Lect 6 Filter, Edge, Detector

Topic 1 — Denoise Image.

Use neighbour pixel's average to replace center point.

eg. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \frac{1}{9}$ or $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \frac{1}{8}$

Formula:

$$(f * g)\underset{\substack{\uparrow \\ \text{at location} (m,n)}}{[m,n]} = \sum_{k,l} f[m-k, n-l]\, g[k,l)$$

For Convolution.

linearity	$f*(g_1+g_2) = f*g_1 + f*g_2$
Shift Invariant	$f[m-t, n-k] * g = (f*g)[m-t, n-k)$
Commutative	$a*b = b*a$
Associative	$(a*b)*c = a*(b*c)$
Distribution	$a*(b+c) = a*b + a*c$
Scalar	$k \cdot a*b = a*kb$
Identy	unit impulse $e = [0 . 0 \cdots 0, 1, 0 \cdots 0]$

Kernals:

① $\frac{1}{\Gamma}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ a reasonable "blurring" kernal, but

-> edge turns abruptly

-> weights are not changed r.s.p. distance.

② Gaussian Kernal

$$G(x.y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Make sure that the kernel is big enough to cover effective points

3-σ rule. to decide the size.

eg. $\sigma = 1$, then, the width of kernel is $(3\times\sigma)\times 2-1=5$

Properties

1. LPF
2. filter 1 with σ_1^2, filter 2 with σ_2^2.

   filter 3 = filter 1 $*$ filter 2 = filter with $(\sigma_1^2+\sigma_2^2)$
3. Separable Kernel

   $\underset{\underset{\text{column}}{\downarrow}}{1D} * \underset{\underset{\text{row}}{\downarrow}}{1D} = 2D$

$$G(x,y) = \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}}\right)$$

Gaussian Filter works for Gaussian noise,
but cannot deal with salt-and-pepper noise properly.
↓
Median Filter

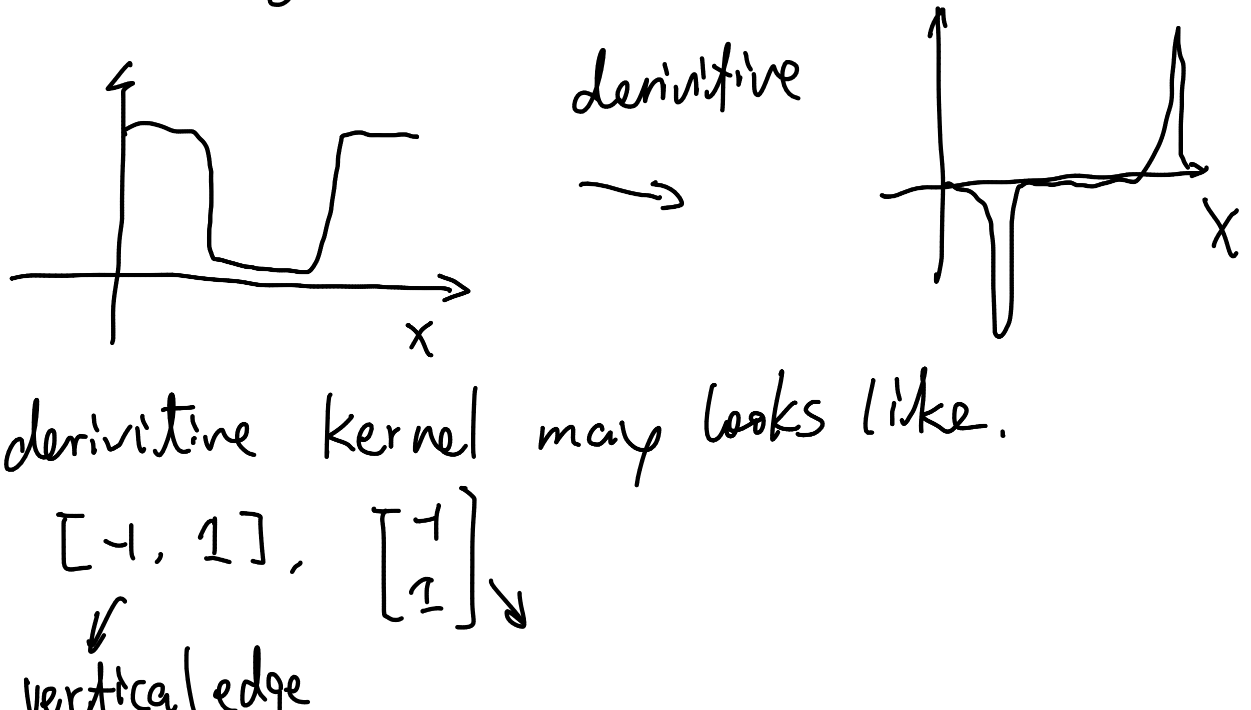
Topic 2 - Edge Detector.

target: Find the place where sudden changes happen.

Causes to edge

1. discontinuity of surface normal
2. depth discontinuity
3. surface color discontinuity
4. illumination discontinuity

For an edge



derivitive kernel may looks like.

$[-1, 1]$, $\begin{bmatrix}-1\\1\end{bmatrix}$ ↘
↙
vertical edge

Some other kernel options:

Prewitt $\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ Sobel $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ Roberts $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Topic 3. Gradient of an Image.

$\nabla f = [\frac{\partial}{\partial x}, \frac{\partial}{\partial y}] f$.

for

vertical line

$\nabla f = [\frac{\partial}{\partial x} f, 0]$

horizontal line

$\nabla f = [0, \frac{\partial}{\partial y} f]$

Gradient's direction is from dark to bright. orthogonal to edge.

Define gradient angle

$$\theta \triangleq \tan^{-1}\left\{\left[\frac{\partial}{\partial y} / \frac{\partial}{\partial x}\right] f\right\}$$

magnitude of grad

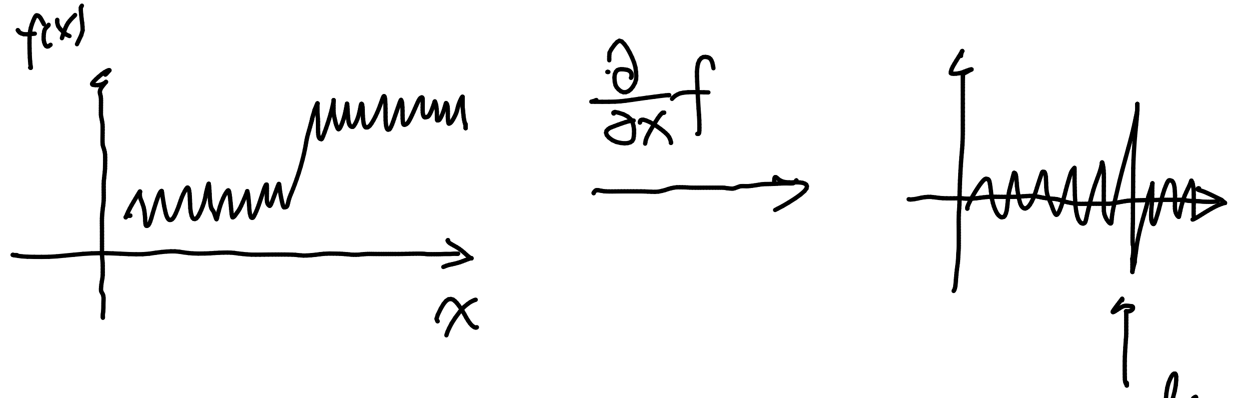
$$\|\nabla f\|_2$$

Application:

If we would like to combine two images. copying gradient instead of copying pixel is a more natural method

Effort of noise.

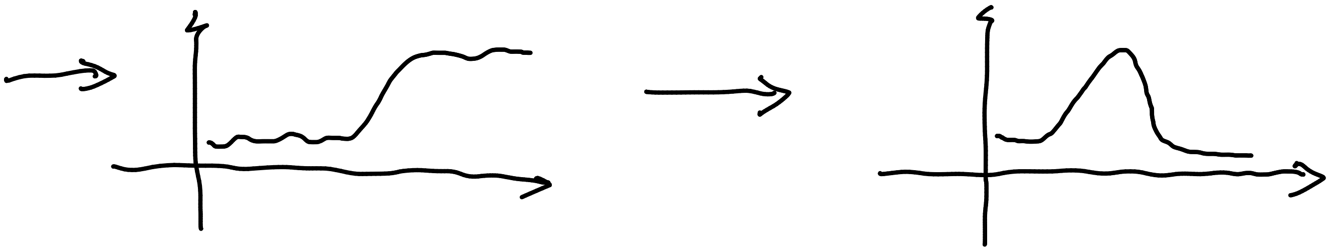
Suppose our image is like this



edge is not obvious.

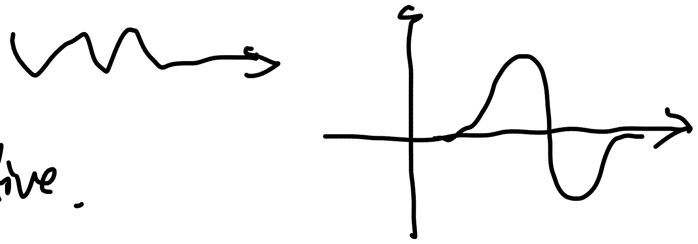
Method:

1. Gaussian Filter (Smooth) 2. Derivitive.



We can build a compond filter

$$f' = \underset{\substack{\uparrow \\ \text{Smooth}}}{f} * \underset{\substack{\nwarrow \\ \text{derivitive.}}}{g}$$



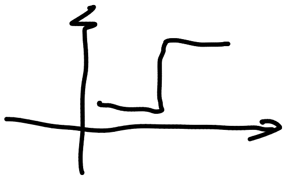
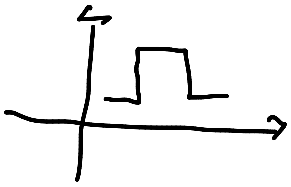
it's the derivitive of Gaussian. still separable.

Canny Edge Detector

A very mature edge detector.

Assumptions: 1. Edge has no width.

2. Edge detector should return True/False, not a low/high value.

Edge is  , if you see  . two edges.

Method for Canny Filter

1. Compute gradient
2. Use a threshold (high) to polarize pixels ↗ Edge
   ↘ Non-Edge
3. Only choose pixel which is maximum among its surroundi
   [non-maximum suppression]
   Interpolation may be applied.
4. Sometimes, edges may not be continuous. So, we then choose a relatively low threshold to continue the edge.

E.g.